

# Binary Bingo

Ico Doornekamp

March 29, 2006

## Abstract

This document specifies a binary algorithm for the distribution of prizes by lot or chance. The algorithm ensures an equal chance of winning prizes for all participants. This document is released under the [GNU Free Documentation License](#).



Figure 1: Heads and tails values

## 1 Rules

1. All participants are assigned a unique number, starting at the number zero, counting up. The number zero will be assigned to the game leader, and from there on numbers are assigned clockwise to the other members of the group. The assigned numbers are known to all participants.
2. The numbers are converted to binary notation, where the MSB shall be on the left, and the LSB on the right side.  
For example, the number 6 will be converted to binary 110.
3. All numbers are padded with zeroes on the left side until all numbers have an equal number of bits.
4. The first prize to be raffled is shown to the participants.
5. A bitstream will be generated by repeatedly flipping a coin. (See appendix A for the coin flipping procedure) The top side of the coin determines the next bit value in the bitstream: *heads* adds a 1 (one) and *tails* adds a 0 (zero) to the end of the bitstream.
6. As soon as the last (rightmost) bits of the generated bitstream match the binary-notated

number of one of the participants, he or she wins the current prize.

7. The next prize is taken from the pool, and the flipping of the coin continues. The new bits are appended to the existing bitstream. Step 6 and 7 are repeated until all prizes have been raffled.

## 2 Game duration

The average duration of the game depends on two factors: the number of participants  $n$  and the number of prizes  $p$ . If  $b$  is the number of bits used for the representation of the participants numbers, the average number of flips to finish the game will be  $b + p * \frac{2^b}{n} - 1$

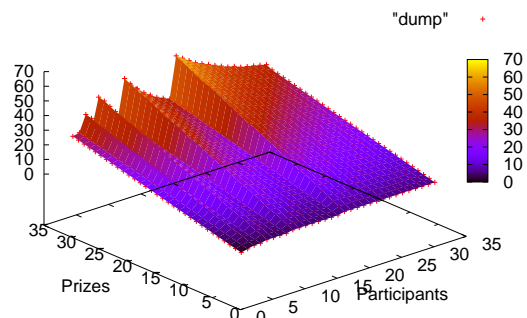


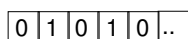
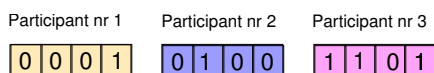
Figure 2: Average number of coin-flips for a game for a number of participants and prizes

No matches will occur until at least  $b$  bits are flipped. After that, every bit generates a new number between 0 and  $2^b$ . The chance of a match with one of the participants numbers is  $\frac{n}{2^b}$  for each new bit added to the bitstream. This means that a match will occur every  $\frac{2^b}{n}$  bits, and this will have to be repeated  $p$  times for all prizes to be raffled. Thus, the total number of flips will be  $b + p * \frac{2^b}{n} - 1$  on average.

The game will be most effective when the number of participants is an exact power of two; in this case a match will occur with every flip of the coin and the total number of flips will be just  $b + p$ .

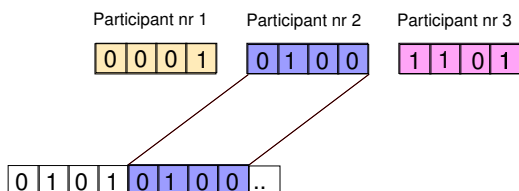
### 3 Example game

Here a part of an example game is given. This example game has between 9 and 16 participants, so the binary notation of the participant numbers takes 4 bits each. This example shows only the beginning of a game, and for simplicity, only 3 participants are shown.



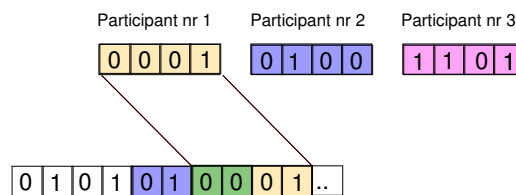
Generated bitstream

The first bits are generated. No matches can occur in the first three bits because the bitstream is not as long as the participant numbers.



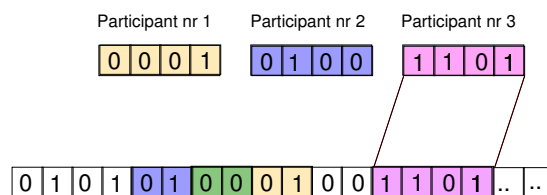
Generated bitstream

Here the last 4 bits match participant #2's binary number. This participant wins the first prize, and the flipping continues.



Generated bitstream

Participant #1's number matches. Note how this number overlaps the previous match of #2.



Generated bitstream

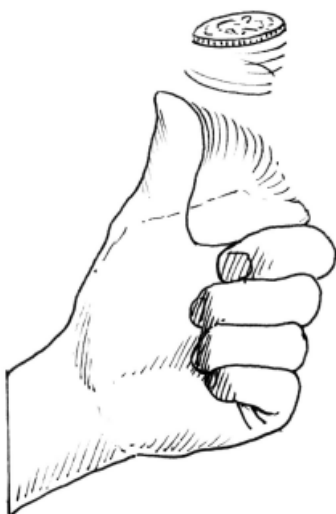
Finally, participant #3's pattern matches. The game still continues until all prizes are distributed.

## A Coin flipping

During coin flipping the coin is "flipped into the air", i.e., caused to both rise and rotate about an axis parallel to its flat surfaces. The coin is flipped by the game leader, performing the following steps:

- resting the coin on the sides of several segments of the bent index finger of the dominant hand,
- pressing the tip of the bent thumb of the same hand against the palm-side of the index finger, so that friction there holds the thumb back from extending further,
- tensing the muscles that extend the thumb, thereby storing energy in the form of tension in those muscles,
- further extending the thumb and/or slightly uncurling the index finger, thereby overcoming the finger's frictional grip against the thumb-tip so it slips, and freely and rapidly extends, with it or its nail hitting the bottom face of the coin, centered within the half of the coin that is less in contact with the bend index finger, and thus simultaneously pushing it more or less upward and setting it rotating around an axis parallel to the circular faces of the coin;

- optionally, suddenly raising and quickly stopping the hand involved, in coordination with the releasing of the thumb, thus imparting extra vertical momentum (but little additional rotary momentum) to the coin. (Depending on the skill of the coin-tosser, and any resulting horizontal motion, the optional upward jerk of the tossing hand may be needed to ensure the coin stays aloft long enough to get the catching hand into position, or for the tosser and observers to move out of its path.);



- once it falls back to a convenient height, bringing one hand down over it, to prevent its bouncing away, as it lands on the other hand or arm, and quickly removing the upper hand from it
- all those involved jointly observing whether it has landed "showing heads" –with the side bearing the portrait or profile uppermost – or "showing tails".

Online flipping services are available at <http://www.random.org/flip.html> and <http://shazam.econ.ubc.ca/flip/>

Binary	Decimal
0	00000
1	00001
2	00010
3	00011
4	00100
5	00101
6	00110
7	00111
8	01000
9	01001
10	01010
11	01011
12	01100
13	01101
14	01110
15	01111
16	10000
17	10001
18	10010
19	10011
20	10100
21	10101
22	10110
23	10111
24	11000
25	11001
26	11010
27	11011
28	11100
29	11101
30	11110
31	11111

Table 1: Binary translation table

## B Example coin bitstream

This section shows an example bitstream that could have resulted from repetitive coin tossing:

00101000100011000110111010010111001110  
00100000100101000010000000110010001111  
0011111110111111111111111111111111101  
0011111111111111111111111111111111110  
0011011101111111101111111111111101110  
0111000101101110111101000001111111111  
01101110110100110010000011110010000110  
00101000001000001010111010000011101100  
01000111110010000001010100001011111010  
00100000110101100100011100110100011110  
00101100001011100011011111010000001110  
01010010101110100010111110111010011111  
10101001001001010000011110110100101100  
10011010100011110000010111110111110101  
0100010101001110000001011111101111001  
00111010011001000011110000111000001011  
01100110011000010010111001000101010111  
10100010011011101000010100011111111001  
0011111111111011001111111111110011100  
0110111111111011101111111011111111100  
111111111111111111111111111111111110  
10110000100110101111011000101110100100  
11111101011011010111110011011110011110  
0101110111111111111110101010001101111  
11010010000111111011100111101000000110  
01111001111111111111011011000111011101  
001001111111111111100110010011001001101  
010111111011111110100011110010000001101  
001111111111111010110111110101000111000  
1011111110011010101001111111101111000  
00110101110110001000000111110111110101  
00100101110000001101100001111111100101  
1000011100001010111100110010101110011  
00001101000100001010011100101110011010  
0100110010000000010011100010101111110  
0101000011111111111111111111111011101  
0110001111111101011111111111101001101  
11111111111111111000111110111111111110  
01101101011111111010000100011100001010  
00011111111000001001100001000010100110  
00111100001000100101001000100010011000  
10111000010100011100001001101000010001  
00011110000000000111100001101000100000  
11101000011011010001110000000100000011  
10011010100100000111000000101010000000  
00011010100100110110000100010011001101  
01101101010000000111000110010100000110  
1011011011100000000000011000110000101  
110101011111110111111111111111111101  
11010101001011111111111010011110111100

00010111010100100010101110001100000111  
00100101001100010010111100101110100101  
00010100010101100111110101111011111000  
00101001001010001000110011110010011101  
01000010100000001110010101110100111100  
01110010110011000111111100011011111100  
00010100101110000111000010000000010100  
10100011001000010110010001010000001110  
10111010001000010010011001000100001110  
00111111111111111110111111111111111111  
011111111111111111111111101110101111111  
001111111111111110111111111111101111111  
11110000001110100111100100000001100110  
00100010110000011011011000101100100111  
01011100000001100100101111001100011101  
00000011010010000101100110010010000111  
01100100101110011111000011100101011111  
01010100011100001110011111100011111100  
10100100100000111010101110000011111111  
00111111111001100000001100010010010001  
11110011110001010000001011111100001101  
0101111010100010000001101111111100111  
00010100100000011001111111100101110001  
01110011100001011011111111001000011110  
1110010111001100101111111000010011101  
11110001000011010111010111100000011111  
111100101010101000111011101101000111110  
1011101000010000011111111110101000111000  
001100101011011111111111010010010111100  
01111111110011111111101010110111110100  
00101111100111111111000011001011011110  
00010011111111100101101001100001001110  
100100011110111111000101010101000011100  
00111110011000000100100000000001010000  
10111110000101011110111100111011001011  
011111100000000110000000110001111111100  
01011111011000111000001101011100000100  
00000000110001010111000001000010101000  
00100000010100100111110000010001001101  
101111111111111111111111111111101111101  
01101111101101111111111111111011111110  
01101011001001100001110001001111111101  
01011000011100110001100001111111110100  
0100011011100000000011011110001101000  
10011011001000101000111111111110011010  
0101101111000010010111111110111001100  
00101101100010111111111111101010110100  
010100101000111111111111010111110001101

1001010101111011111001001000001010100  
0000001111111110110110110011100110000  
0111111101111101000010000001010001110  
111111111111111111111111111101111100  
001110011011111111111101111001111101  
01000101000111001011101001100000001000  
01000100010011010010000010110010000011  
00100011101110010011111101111010001111  
00101010001010000110000010001000100101  
0011111111111111111111011111101111011  
11111011111110111111111111111111101  
101111111111111110110111110111111111  
11001011001000010000011110000011000110  
01111001110000001001000001011111010101  
00100011000101000111010000100001001111  
01110000010110010110000111011011000010  
00101101011011110110101100110000110100  
01110000010110001011010110100000011000  
00001000000111011110000000101101001001  
11111000010000011101011000001101010011  
01110000100000001100010101110011001100  
11111000011101011010000000000001101000  
10111111101010011110111100101100000110  
10001111101001000011111101011000100000  
00011000000010000101110011111010110001  
010011000000001011100100101010101010001  
00000001000111001101010101001010000100  
10110001000000101001000001100000111111  
00001111001111000001011001001110000001  
11110110101001010110000100000101100100  
10001000110101000010000110101011001011  
00000011100010010101100011001000000000  
00101010001111000010110010010010000001  
1010110010111000000011001110110001100  
10110111010001101011000110010100001010  
01100011111000000000100000110000001100  
00111011100011101011101101100110001100  
11111000100100000110000001011000010000  
10110111000000100000001101000001100101  
10010001101001000010001010101101000110  
1111111111111111111111111111101110  
0111111111111011111111101101111011100  
0111011111111111101111111111100111100  
00100111111000000010011000010010000110  
00110100001010011000011000110000010010  
00010010110000010100010000000110010110  
01010011100001001001111011001100000010  
00010100101101111100111000110110011001  
11100110001100000010011110000001001101  
0011000000000101000101110101101111000

01001100001111001011001111101010111100  
0001101000110001101010111111111111000  
00110100100010001100110111011110111001  
01000000001100000100001110101110000000  
00110110010000001001110100100000101100  
01111101000010101000101010111111100010  
01111110111111111011101111111110111110  
00111111111111111101111111111011111100  
0111111111111011111111111101011111111  
11101000110010011001100101001100100110  
10100000100100000110110100000111001110  
1000110100000011111110111100000000100  
1110000011011011110111100000100100110  
0100101011111111111111101000000001110  
001101111111111111100111010010000001101  
001111111111111010110001100010000111010  
0111111111100000101101101010001111100  
1111011111000011110110111111101111101  
0110111101101001100001011111111111011  
001001110101000001010100110110010001  
00101010111010001100100011000000001100  
11101110010101101100011001010000000100  
10100000110011101010110100000111101001  
0111001011111111101111111111111111110  
1101011111101111101111110111111101100  
10111110010111111111111111110111110100  
001111111111011001100010101010100011111  
011111110100110101111110000000010101100  
00011101000110001000001000001100000001  
10111101001100110000010100101100100011  
01101110101100011100001001011000101110  
01101001000001100110100100000010001010  
10111100000011110011000011100100101011  
11011110001111111000100010010010110011  
01101110000100101111000100010100000000  
01000011100010000110001100000000000110  
00111101110101011111111011111011111101  
01010010001011011111111111101101111110  
00100101010101011001111101011001011111  
01010001101101000001001110100110000100  
10100100111111010000010100010011111101  
01101000011101001110010111101111110001  
00100000100000100110101000010010011110  
00010000101000101010111001000100011100  
01101111100101110010000001100001011101  
11101000000010111010000110000001101100  
10100110000100000000100011000110010000  
1011111111111111111011111111111111101  
1111111111111011111001111111111111000  
001111011111111110111110111101011101101

00101100010101101000010011000000000010  
01110101011101000010101011101101000100  
1111110101001001000000001000110111101  
10001000000101111000000111001111001110  
00000000100111010011100001100010101110  
10011101011110001000110010010011111101  
11101000000011010101011011011111111111  
0011111100100010100100101001100001010  
00010111110000010010001011111101010001  
10011111010001000100111111111111111010  
01011001000010110001111111010101111110  
0011011100000111110111110000000011000  
01010000110110000011111100101000011100  
01100000001010000011111101100010100111  
00100000101010000111011001001110011101  
10010000101011000111010100011011010101  
10111001100000100111111101110011101110  
0101101100000111111110110100000010110  
00011111010111110111010000011011110010  
0001111111111111111000100010011111111  
0110001111111111101101100010010010100  
10101001111011110000001000111100001000  
00011101011100100000101110011110001001  
10111110001010010000100100000111010000  
1011111110011101101000110000010001000  
10110101111011111100011100010000000110  
0111010000011111010100011010110011110  
11100101000000001010010001101100100100  
00100000100010001000111010110110010110  
0111111100111010111111011111110111100  
0011111111111111111111111111111111100  
00110001000001000011010110111111111110  
01110000000001100010100100111111011100  
1000001010000000101011011111111110110  
11110101111111000111101111111111110001  
01100011100101101111111111110100011001  
00001010010100011111101111000000001010  
11001101100111111111111110011110000100  
00101101111111111111100000000011000000  
0111100111111111111010100101010011000  
00011111111110111111100100000010001110  
011111110111111111101111011011111100  
0011101111111111101111111111111111111  
10011100001010000000001000011100011101  
0101111100000110000101010110000011111  
10101011001000001101001000000010001111  
01000100001100100110111100011011011110  
0011110111111111111111111111111111101  
011111111111110101111111101111111110  
0111111010111111101110011111111111100

11110011010111000111100010110011000101  
0110000000001110100010000000000001110  
00100100000010011011001111000110110001  
10100000001100011100011001000111011110  
01100100010000110101001000100000010000  
01100000001010100011101010000110000000  
01110001011100001111011110001111011100  
00010000101000001111110011000001101110  
01111110011110001010001000101000001000  
10111100001000101101011001101101000011  
00101111101011100100000100000110010010  
10000010010111000001101011111011100010  
00100110100010011001100001010110111100  
00101100111101000111101000010111000101  
10000001110011000101111011010001010011  
01011001010000000010011000101011010111  
10010001000000011100001010010011011011  
01101100110010011011001101000011000101  
00001001000100000111011101111001001110  
01010000010010010101101110011000000001  
01011101100001000010011110000110011100  
001100000101010110000010100001001001  
10110000011010010110000010100001001001  
000110000000111000000011001000110011  
11011111000001101001100110010011101000  
0110010010001100001101000111110000000001  
101001100010001101000111000000010100111  
00001101011110010100110100001000000100  
11101000000010000010100101000000100101  
11011010001110010000011011110111001111  
10000100001101011010011000001000011110  
000011101101101011011111110110011100  
00010100000010100010111100000010111100  
00101000011111001100101111111111111000  
0011001100101101101000011111110111000  
00000011110001101111100000111110101010  
10000101000010100100001100001011010110  
11100110010111100000110000101011000110  
0111111101111111111111111111111111111  
00111111111100100111111111111111111000  
0111011111111111111111111111111111110  
00111000101000100111100100101001010100  
11100000101100010111110001101100011111  
11010111000000111111110011000001010100

0001001101011111111110101001000000101  
01001000111101101010001111011011001100  
0011011111111101110001111100000101111  
0011110111011100010001111100011111111  
01011110111001010100101110111010111101  
1011111001000010001101111111111111000  
0011111010110110110110111011111111000  
00100001001000010010001001111110000000  
00100010011101100010001010000001101010  
00110110110000010001101000010010010101  
01111101000011010110110110101000101110  
0100001101111110111111111111111111110  
101000111111111111011111111111111101  
0101011101111111111111111111111111110  
00101011011011000010110111000010000111  
11011011001000110000001000001010010100  
11111101010100100001100100010010100001  
10110000110100011000000110100010010100  
00011000110001110001000010010111001000  
00111001000100101001001101101101001000  
10111000001100100110000001000000100001  
00010110001000000001010011000011100111  
01100110011100101111001101011100101010  
11000111000101010010011000100001101101  
00010011110111111110101111111111111100  
0100011010011110111111111111111111101  
00011001110000001011010010100000010100  
00011011100110000101111100000010100011  
00010001000100100101000010110011101100  
00000001110010001000010110001011111100  
10000000100000011001100000010101111111  
00100000011011010011001000101100101110  
01000000100000101000110000000101101110  
10100001001100100100000110010110111110  
10101010001010010010010011000001110100  
1111111110100111111111111111111111101  
111111101111011101101111010111011100  
0011011011111111111111111111111011101  
10100110101110100100100010100100010110  
11111110011011010101010100011010110110  
00001001111001111010011000001110001110  
00110010010010110000110000111110011000  
10011100101001010001001001101111001101  
10110000111100000010111101001111111100  
11001101011010000100000001110111011001  
10101011101010001010110010110100001000  
00111110010100100101011110101111001001  
0101110100001101110101111111110100101  
00010011011001010110111110110000111101  
001111110011111010111100000101101101

001000000000000001111110011111101101  
11101100100110000011111110010010101110  
00110111001111111011000100010010100110  
11111000100010001111101101000101001100  
10111100101000001111110110010101001101  
11011010011011011111110001001101011100  
01011110001011111110000000010011111011  
11101111111111111011000001000010101110  
1011001111111111111100001100100010000  
10010010111111110011100110101010011010  
0111110101100000000001001101100000100  
0101111011101000000101010100110000101  
00110110000001101010000100110000010000  
010111101100010010011001010110110100  
01101101101001011001111101011001100001  
11110010000010000000010100110100001011  
00110001101001001000010100001100110101  
101111111111111111111111101111011010  
1011111010111111111111111111111111100  
10100001011001001010001101010111111110  
10010010010101000010000100111111111101  
00010100101011000100000011111110111011  
01001001011000101111110111111011110110  
01011100100011100111111111110010010111  
00100100000000110011111111001000100011  
0011111100011111111111111111111000100100  
110111110111111111111111110010000111010  
010010011111111111110000001011110010000  
000011111101111111111100100000100010110  
10111011111111111111111111111111111100  
01111111111111111111111111111111111110  
00111000110000101001001110011000010101  
10000011110011110000100010000000110101  
00100110010000011100100100111000100101  
010011101101100010000000000001001010101  
001111111110111111111111111111100111100  
11111111111111111111111111111111111100  
01111111111111111111111111111111111111  
11111001101100100101000000110000100110  
1110010110110101010000100001000100000111  
01101111000011000001011100110100000111  
10111100100011110000000010001010000000  
01111011011100010100101110110010001010  
00100000110010010010100010011111110010  
00110111001111111000001001110011110100  
10101011000010000000001101110000110101  
10110100000011010000000110010011001000  
10111010001011011000011110010001010000  
00111110110000100010101100110110001000  
1110100010000000001011110000101000011

00010100011111101010001101011001011011  
11000000011100110011100000010110101000  
00010001111011001110000010101100001101  
10000001001101000001111000000101001001  
00101000000110011000110001001001010010  
00010011000011001000010011010010010010  
01000001110011000110000111010010100101  
00011110010010010000110010111000110001  
01000010001010010011010010101000110100  
00000100010110110010000110001100101001  
01101110001010011101100110111010001101  
11000001100011000100001011111100100001  
00001100100101011001010100000110110000  
00001100101000100100010010001000010010  
00100010010001110001010001101010111100  
00100000100010111101001110111011101100  
101110111111111011111111111111111100  
0011111111111111111110111011101111111  
111111010111011110111111111111011011  
11111101011101010010000010100001001111  
10100010000110000100011100010100001100  
01111011000100101100110000000110000101  
01011000010010000011100010100100010101  
00001110001100001100011010001011001100  
10000101001010101100011101011001101110  
01011100000111100101011111110000011100  
01011000000100100010011110111010101010  
0110000111100011000000011110111111010  
0001000001101000011010111111111111010  
01001000000110101101000001011110111000  
00110100010001001100100101000100001100  
00110011010000101010111110000000001001  
011101110111111111111111011111110111  
0111111111111111111110111111111111111  
1011101111110111111111111111111111110  
10110010101001010111110101111000000110  
00101011010110110111011100010000011111  
10001010110110111111111100001110010101  
00000000000111101111111110011100010100  
0000101111111111110011011010101011111  
1010011011111111100011111101011111101  
00111111111110101000011100001111101101  
11111111111000100100011110000101111100  
10010111111101100000111111111101101110  
01101111011000000000011101111111101110  
00100010000100011010000000111110010010  
01101000011011111001110100110101100100  
01001000101100110000000011000010001110  
00000000000100011110000001110000000101  
1000000011111101111011111111111111100

01000011111111111111111111111111111101  
000101111111101111111111111011111111111  
000111111111000100100001001101000001100  
00111011011101001000010110000011010100  
11011100100001110100000000000101000100  
101010010001011001010011111100010110100  
10111000111110110001010000011001110111  
10110100110100100001101001010100111001  
10111111000101000011011011100101011000  
10011010001000110101001001001000101000  
00001111000100011000001001001000110100  
0000111111000000001001101111100011010  
10001001111111111111111111110111101100  
10110010010111011011011111111111111111  
01001001101000001010000011111000100100  
00101011011101100001100110101111010100  
10100101011100110010111000000011111101  
00000100110101001010101000100111111110  
00000010010000100111000000000100011110  
10110101111001011000011100010000001110  
10001000001000000000010000011100100100  
0110100110010011101010100100000010110  
11110001100101111000010000001110010010  
01011111111111101101101111111111111100  
11111110111111111001111001111111111110  
1111011100010110001000100010111111100  
10110001000000111010001000011011010111  
10000000000100001011011010111101110101  
00000000101000100100100000101101001111  
0001000000000010101000000111011001100  
11001100010001101011100100100011111101  
11000011100001001001100100001011111100  
0111111110010000101010000000100000000  
11111111010001011001010111111110101001  
10011111010010001101011111011111110000  
010110000001001010000111111010001111011  
00110110011100011001110101101010111011  
01111010111000111011111101001101001100  
00100111110011000011111100001100011110  
10110111001000001111111001000010111100  
11110001000100010111111110100100000101  
11110111011110001111110001010101111001  
00111111100101111111011101100010111100  
10110101100101111111000101100111111101  
1000111111110111111001001010111101101  
0111001111111101100000010101100000100  
00010010111111010010001000110100011100  
01011111110010000000101010001101010000  
00111110111100101101101000100010000010



00110110101100010100110100001100100000  
10111110100000010001100111000110100001  
11010010000110101001011010010100101101  
10001110011001001010000001010010010100  
01101000001101000000000010000000010111  
0011011111111111101111111111111111110  
11111111111111111111111111101111111100  
10100101000101100001000001000111111100  
00000010010001010001101011111111111110  
01000000001100010000001111101111111110  
0000001100000100000011111111111000101  
1101010100000110011111111110010100001  
01101100000100011111110111001010111001  
1011100100111111111111010101101000001  
0110000100111111111001101100001000011  
01111001111111011110100011010000000100  
1000111111111100000001001010111111111  
0111111111111101111011111111111111100  
1111001111111111111111111111101111111  
1000000001001011111100110001100010100  
0001000111110011101110010010011101001  
00100000011000100000100010000000000101  
11111001100110111010101101000011000110  
10111111011011111101111101111111111100  
0011111111011111110111011111111111100  
00111111111110111111001111111111111001  
10100100001000001001000010001001100101  
10101111000010001100100101010100000110  
00111101001100001011001110101000010100  
01110001111000000000110001111000001101  
10111010100001000011001110001000000100  
00110010000110010010100011110100000100  
1111000100001101011111001110100010111  
00100100010001001101100100011101001010  
00101011100000100000010001000000100111  
0111101000010001000000100100101000010  
00111110100100010100000000111000000111  
01111101000110000100101011001011011001  
00010010000000110001101111000010011001  
00100110100011100011000001100001010010  
00000011101000100001111000011111000000  
01001110011100101101010100011111001001  
10000001001000000100011010010000010110  
00010101001100011001011101000000001010  
01101000001011100010000000110001000110  
10100110010001010101000001100001010100  
11001110000111010000000001100011000011  
10011011100100001000001011010110111001  
01100001011001011010101011110010101001  
00010000110110010110000100100111100010

01100001111001100000010010010001110010  
11010101010010110001010001101111100000  
10101001011000100010011011000100010111  
10101100100011011011001000000000100110  
01111111111111111111111110111111011100  
01111101110111111110011100111111101100  
1011111111101111110011111101111111000  
10101001100011101000101000000010011110  
11010010011001101110010101000001010110  
01100100010111000100011011011110001101  
10111000001101100010010010101011000100  
0010000110010000110011101001000001000  
00100101001000000000111110000101001100  
10001100110111010000111101010110010111  
00110111000101001010001111000110111111  
01010100001100000111001111100111111001  
010011010001010000100101111111111000011  
00010000010000010100100000111111001001  
01110110100000010011110001110110100100  
00101001001111010000001010001000110101  
10111111001111001111011111111111111101  
10111111111111111111111111111111111100  
00011110111111111111111011110111111111  
10110010100011000011011111100000000111  
00100100010011101101010010000011000100  
001100011000101111111011100110010000110  
01001011010111111111111110001011101100  
0011111111110110010111100011110111100  
1111111110000010100000111111101111000  
10101100110101000001100111111110110000  
00001101010110110010000001111110110100  
00111001000010101000000010000100110001  
11100000101111110110000000100010000100  
10101100010000011011001001101011011110  
010010000011111111111101101111111011101  
10111001111110110111011111111011111111  
010001111111111111111111111110111011100  
00011111010000000001011100110010100100  
10011111000010111101010101101111100100  
01111110000010100010000100010001001001  
00111010100010001110100010001100111110  
10111000100011010011111000110000010101  
00110010001101000000001011001001000101  
10110100010011000110110000011010100110  
10111010111001100001101001000000000001  
10101101000000010100000100010110101000  
110011111001010010100100110100111100

001101011111111101101111111101111110  
100010001010101011111111111111110101  
10000100111001010100101101110001111111  
10110011100010000110111101100100010100  
00001101011110010010010100100011111010  
01110001111000010101101101001111111100  
01100000011001001110011101001111011100  
00100010101010000111100000010100101110  
01110101110100011000110011111000101101  
11011101001001101010101010101000001100  
0110000000001101111000100100001100100  
0110111111111111111011111111111111111  
011111111111111111111111111110110110  
11111011111111111111111111111111101  
11101001101110000000000110000100010101  
00100101001110010001000010001000001110  
10001000110010000000000010000000000101  
10011000000010010011000101000010101100  
11110001010001110111001011101001001100  
10001000010000000011000110000111110110  
10010101011110010101110011001011111100  
01111011001111100110100000110001101000  
01111111110000101001000010111101101110  
00011111010011101000011100011101101111  
01011010001001100100011101101001110010  
11111010010000010001111110000000011010  
10101000100100010111110100101000101101  
10111001010000001111011010010010000100  
00100101101100110111110001010110000101  
10111010010011000111111010100000111010  
10111000100001001101010000101011101110  
01111101011100011011111100001000011110  
00111111001011011111100000010011110010  
0000101101011111111011000011011111101  
0100001111110111101010111100001000000  
1110111111011111001001100100110100110  
00110101011100110101010001100110110001  
01011111000111001000000110011011101010  
10111110010101101010101010101101001001  
01011110101100011100001011010000011111  
00010011000010100100000011001110000101  
01100001100000100010101000000000100100  
10110010001010010010100010010110001100  
0111111011110001111111111111111111111  
10111111011111111111101111101110111110  
11111100010001010001100011000111011110  
10011100010000000000111101111101111101  
10011101110010100111001111110111011000  
00110110101101111010111111011111001101  
0111010100001100110111111111010110001

10100100000111111101110110001010101000  
00000000011111110111111001000000011000  
00101010111111110110111100010000000100  
10100111010111110101001000000101110101  
10011111111111101000000100000110110010  
111111111111111110111111111111111101  
01111100111111011111111111110111111100  
00110111000000000000110000001011000100  
10000010001110010000001101101100000000  
01011010011100101110000001110000000111  
10110010000001101000111000110101001110  
01111111111011111111101011111111011101  
01111111111111111111111111111111111100  
00111110111111111111111011111111111100  
11110001010001010011011000001010001010  
00111000001100111100010000000101000100  
01110100011000100101000000000110000011  
00001110101001010100000010010010000001  
0010010001001001110000011101010000111  
00100010110011011001001001010001101111  
10101001110001100011010100000010010100  
10101110101000010101001100010111101101  
11110000101011101001000010100100000111  
01111000001101111001011011011111000001  
0011111100000100111100000101000101101  
10000111100010100000000111011001101000